achromat

achromat: A usually two-element lens that is corrected to bring two specified or distinct wavelengths to a common focal point. *Note 1:* The term *"achromatic"* literally means "without color." This is not strictly true, however. Early lenses consisted of only a single element, and therefore could bring only a single wavelength to a given focal point; *i.e.*, they suffered from what is termed *"chromatic aberration."* The invention of lenses with two elements meant that two distinct wavelengths could be brought to a common focus. This represented a vast improvement over the single-element lens; hence the designation *"achromat(ic)." Note 2:* The residual chromatic aberration manifested in the image produced by an achromat (and other multi-element lenses) is usually referred to as the *"secondary spectrum." Synonyms* **achromatic doublet, achromatic lens.**

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be rearranged to say that the ray angle magnification condition takes the form

$$\frac{\sin \theta'}{\sin \theta} = \frac{nx}{n'x'} = \frac{n}{n'm_x} = \text{const.} \tag{4.104}$$

Here $m_x = x'/x$ is the transverse magnification. Thus, if a series of rays leaves P at the angles $\theta_1, \theta_2, \ldots$ and arrives at P' at the angles $\theta'_1, \theta'_2, \ldots$, we must have

$$\frac{\sin \theta'_1}{\sin \theta_1} = \frac{\sin \theta'_2}{\sin \theta_2} = \ldots \tag{4.105}$$

Equation (4.105) can be used to test for an aplanatic system, that is, one with perfect imaging of slightly off-axis points. Interestingly, this test involves only the behavior of rays from an on-axis point.

The sine condition has already been used to derive Eq. (4.39), the law of conservation of luminance or radiance in an elementary beam. It is possible to derive this conservation law from the laws of thermodynamics and then derive the sine condition from the conservation law.

Real optical systems can be made aplanatic or nearly aplanatic by a variety of devices such as the use of meniscus lenses. They will be useful only for small off-axis displacements and only for a single object plane. They can be used in microscopes and certain telescopes where the field of view is small and the object plane is fixed. For wider fields of view the aberrations that are quadratic or higher in x' become important, and one cannot reduce them in general without giving up the sine condition and accepting some coma and related effects.

B. Chromatic Aberrations

Because most optical systems are used with light of varying wavelengths, the dispersions of the refracting media must be taken into account. The optical designer must consider the variation of refractive indices with wavelength so that the resulting performance of the system is more or less wavelength independent.

We can distinguish two general types of chromatic aberrations: (1) chromatic variations in the paraxial image-forming properties of a system, and (2) wavelength dependence of the monochromatic aberrations. The second type is evaluated by treating the refractive indices as variable in section A of this chapter. We will concentrate here primarily on the first type. With that in mind, we return to the paraxial imaging conditions.

In paraxial optics the properties of an image-forming system depend solely on the locations of the principal planes and the focal planes. Chromatic aberrations result when the position of any of these planes is wavelength dependent. We can discuss the effects of monochromatic aberrations in terms of a variation in image distance along the axis and a variation in transverse magnification. Because a single-refracting surface shows behavior that can be observed in more complex systems, we will start with it.

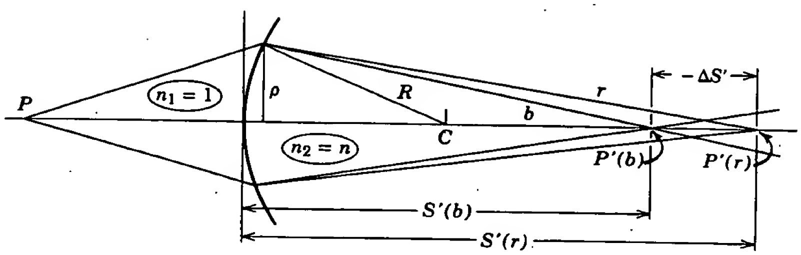

Fig. 4.47 When chromatic aberration is present, the paraxial image point depends on the wavelength of the light. Blue light (b) and red light (r) create different images that are separated by $\Delta S'$.

1. Single-Refracting Surface. Consider first an object point P on the axis of the system. With refractive indices as shown in Fig. 4.47, the image formation condition is

$$\frac{1}{S} + \frac{n}{S'} = \frac{n-1}{R} \tag{4.106}$$

Let us denote two different wavelengths by r and b (for red and blue, typically) and assume that the indices have the property $n(b) > n(r)$. The change in index $\Delta n = n(b) - n(r)$ will be small compared with unity for transparent optical media, and we may use approximations to first order in Δn. Then we let $\Delta S' = S'(b) - S'(r)$ represent the longitudinal chromatic aberration, and by differentiating Eq. (4.106) we obtain

$$\Delta\left(\frac{1}{S'}\right) = -\frac{\Delta S'}{S'^2} = \frac{\Delta n}{n}\left(\frac{1}{R} - \frac{1}{S'}\right) \tag{4.107}$$

Here n and S' can be evaluated at any convenient wavelength, but best accuracy is obtained with a wavelength between r and b.

At $P'(b)$, the focus for b light, the rays of r light will form a cone converging toward $P'(r)$. The cone will have a half-angle of approximately ρ/S', and it will intersect the b image plane in a disk of radius

$$\Delta x' = -\Delta S' \frac{\rho}{S'} = S'\rho \frac{\Delta n}{n}\left(\frac{1}{R} - \frac{1}{S'}\right) \tag{4.108}$$

A disk of approximately the same radius is formed by b rays in the r image plane. This is shown in Fig. 4.48.

An off-axis object point will produce images in r and b light at different heights from the axis. This is shown in Fig. 4.49. The object point P has chromatic paraxial point images at $P'(b)$ and $P'(r)$ that lie along the undeviated ray (UR) that passes through the center of curvature at C. This leads to a wavelength-dependent transverse magnification.

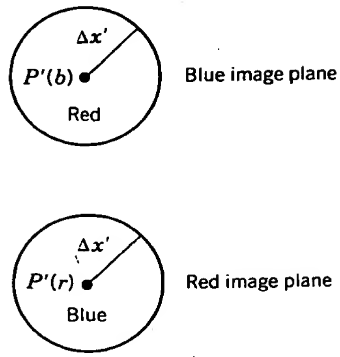

Fig. 4.48 Appearance of the "focus" in the vicinity of the paraxial image points for blue and red light.

The fractional change in magnification

$$\frac{\Delta m_x}{m_x} = \frac{x'(b) - x'(r)}{x'} \tag{4.109}$$

where x' is the average of $x'(b)$ and $x'(r)$, can be found through a consideration of the similar triangles shown in the inset of Fig. 4.49. We have

$$\frac{x'}{(S' - R)} = \frac{-[x'(b) - x'(r)]}{\Delta S'}$$

so that

$$\frac{\Delta S'}{(S' - R)} = \frac{x'(b) - x'(r)}{x'} = \frac{\Delta m_x}{m_x} \tag{4.110}$$

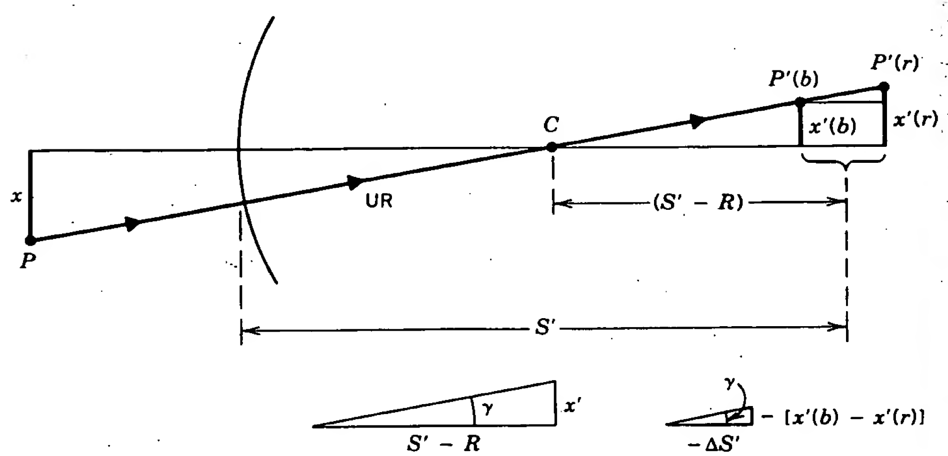

Fig. 4.49 Chromatic dependence of the position of the paraxial focus in a direction perpendicular to the optical axis.

This can be modified by rewriting Eq. (4.107) as

$$\frac{\Delta S'}{S'^2} = -\frac{\Delta n}{n}\frac{(S' - R)}{RS'}$$

leading to

$$\frac{\Delta S'}{(S' - R)} = -\frac{\Delta n}{n}\frac{S'}{R} \tag{4.111}$$

Comparing this to Eq. (4.110) yields the final result

$$\frac{\Delta m_x}{m_x} = -\frac{\Delta n}{n}\frac{S'}{R} \tag{4.112}$$

which is independent of x'.

In complex systems with several refracting surfaces we can often remove the longitudinal chromatic aberration so that the blue and red images coincide for an on-axis object point at a particular distance. Unless the different colored off-axis rays themselves coincide, in general, there will still be chromatic aberration for off-axis object points. Thus the wavelength dependent transverse magnification is harder to correct.

2. Lenses in Contact. The cemented doublet is one of the simplest and most common of the "achromatic" designs. Given two thin lenses with essentially zero separation d, we can write the power of the combination from Eq. (3.79) as

$$\frac{1}{f} = \mathscr{P} = \mathscr{P}_1 + \mathscr{P}_2$$

with

$$\mathscr{P}_1 = \frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R'_1}\right) \equiv (n_1 - 1)K_1 \tag{4.113a}$$

and

$$\mathscr{P}_2 = \frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_2} - \frac{1}{R'_2}\right) \equiv (n_2 - 1)K_2 \tag{4.113b}$$

The quantities K_1, K_2 are called the *total curvature* of each lens. The change in total power caused by a change in wavelength is

$$\begin{aligned} \Delta\mathscr{P} &= \Delta n_1 K_1 + \Delta n_2 K_2 \\ &= \left[\frac{\Delta n_1}{n_1 - 1}\right](n_1 - 1)K_1 + \left[\frac{\Delta n_2}{n_2 - 1}\right](n_2 - 1)K_2 = \frac{\mathscr{P}_1}{V_1} + \frac{\mathscr{P}_2}{V_2} \end{aligned} \tag{4.114}$$

We used Eq. (4.113) in the second equality and have introduced the *dispersion constants* of the glasses in the two lenses, which are defined as follows:

$$V_1 \equiv \frac{n_1 - 1}{\Delta n_1}, \quad V_2 \equiv \frac{n_2 - 1}{\Delta n_2} \tag{4.115}$$

The wavelengths at which the n's and Δn's in Eq. (4.115) are determined are taken conventionally to be the following:

$$\Delta n = n(F) - n(C)$$

$$n = n(D)$$

where F represents the blue line of hydrogen at 486.1 nm, C the red line of hydrogen at 656.3 nm, and D the yellow lines of sodium at 589.3 nm. The D line is close to the wavelength of maximum response of the human eye. The F and C lines gave a convenient spread across the visible spectrum. Data on indices of refraction and dispersion constants are supplied by the glass manufacturers often as a six-digit number; for example, 517645 means $n(D) = 1.517$, $V = 64.5$.

Some black-and-white film has peak sensitivity in the blue region of the spectrum; we would then want to select a camera lens achromatized for shorter wavelengths than the F and C lines.

We return now to Eq. (4.114). For achromatization at the two wavelengths used to define Δn, we set $\Delta\mathscr{P} = 0$ and through use of Eq. (4.115) obtain

$$\frac{\mathscr{P}_2}{\mathscr{P}_1} = -\frac{V_2}{V_1} \tag{4.116}$$

or

$$f_1 V_1 + f_2 V_2 = 0 \tag{4.117}$$

By writing

$$\mathscr{P} = \mathscr{P}_1 + \mathscr{P}_2 = \mathscr{P}_2 - \left(\frac{V_1}{V_2}\right)\mathscr{P}_2$$

we obtain

$$\mathscr{P}_2 = \frac{V_2}{V_2 - V_1}\mathscr{P}$$

and

$$\mathscr{P}_1 = \frac{-V_1}{V_2 - V_1}\mathscr{P} \tag{4.118}$$

Equations (4.118) give the necessary condition for our thin lens to be achromatized for the two wavelengths used to define Δn. The chromatic aberration that remains at other wavelengths is called the *secondary spectrum* and is often small enough to be neglected.

Note that $\mathscr{P}_1$ and $\mathscr{P}_2$ must have opposite signs. If $\mathscr{P}$ is to be positive, we need $\mathscr{P}_1 > -\mathscr{P}_2$, assuming that $\mathscr{P}_1$ is positive. Then by Eq. (4.116) we want

$$\frac{V_1}{V_2} = \frac{\mathscr{P}_1}{\mathscr{P}_2} > 1$$

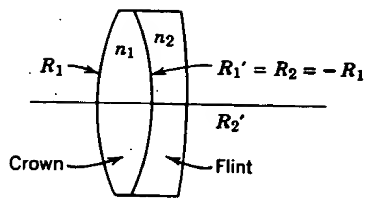

Fig. 4.50 Achromatic lens.

Thus the positive component must have the larger dispersion constant (and hence the smaller dispersion Δn).

To control the secondary spectrum and to minimize the monochromatic aberrations, we want to keep the magnitudes of $\mathscr{P}_1$ and $\mathscr{P}_2$ small. This can be done if $|V_2 - V_1|$ is large.

The shape factors of the individual lenses do not influence Eqs. (4.118). These can be varied to reduce spherical aberration and coma. For inexpensive achromatic doublets, the shape factors are more likely to be chosen for ease of manufacture. For instance, the positive element is often equiconvex; the negative element then becomes almost planoconcave for typical glasses used in a cemented doublet.

As an example, consider a cemented doublet made of borosilicate crown glass No. 517645, $n(D) = 1.517$, $V = 64.5$, as the positive element and dense flint glass No. 617366 as the negative element. Then if the focal length is to be 10 cm = 0.1 m, the power $\mathscr{P}$ will be $10\ \mathrm{m}^{-1}$ (diopter). Equations (4.118) then give $\mathscr{P}_2 = -13.118$ diopters and $\mathscr{P}_1 = 23.118$ diopters. If the positive element is equiconvex, the radii of curvature then are $R_1 = -R' = 4.47$ cm. Because the elements are in contact, we must have $R_2 = -4.47$ cm. Then we can calculate R_2' to be 91.2 cm. The lens is illustrated in Fig. 4.50.

3. Separated Doublet. The system consisting of two thin lenses separated by a distance d in air is a model for several important optical instruments. If the components have powers of $\mathscr{P}_1$ and $\mathscr{P}_2$ then, from Eq. (3.79), the total power will be

$$\frac{1}{f} = \mathscr{P} = \mathscr{P}_1 + \mathscr{P}_2 - \mathscr{P}_1\mathscr{P}_2 d$$

Then the change in $\mathscr{P}$ at two different wavelengths is given by

$$\Delta\mathscr{P} = \Delta\mathscr{P}_1 + \Delta\mathscr{P}_2 - d(\mathscr{P}_2\,\Delta\mathscr{P}_1 + \mathscr{P}_1\,\Delta\mathscr{P}_2) \tag{4.119}$$

Proceeding as before, we write

$$\mathscr{P}_1 = (n_1 - 1)K_1 = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)$$

$$\mathscr{P}_2 = (n_2 - 1)K_1 = (n_2 - 1)\left(\frac{1}{R_2} - \frac{1}{R_2'}\right)$$